Zero : Jurnal Sains, Matematika, dan Terapan
E-ISSN : 2580-5754
P-ISSN : 2580-569X
Vol. X, No. X, Juni YYYY
pp. 01-09

Analysis of Ruin Probability in Insurance Companies Using the Cramer-Lundberg Model with Variations in Claim Distributions

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Article Info

Article history:

Received mm dd, yyyy
Revised mm dd, yyyy
Accepted mm dd, yyyy

Keywords:

Cramer-Lundberg, Ruin Probability, Monte Carlo, Claim Distribution

ABSTRACT

Ruin risk is a crucial issue in the insurance industry, concerning the company's ability to meet claim obligations in the log term. This study aims to simulate the ruin probability of insurance companies using the Cramer-Lundberg model as the primary framework for modeling fund surplus. The Monde Carlo method is employed to estimate ruin probabilities based on different claim distributions, namely exponential, lognormal, and gamma. Simulations are conducted by varying the initial capital, while keeping the premium rate, claim intensity, and claim distribution parameters constant. The results show that the lognormal distribution yields higher ruin probabilities compared to the exponential and gamma distributions for the same initial capital levels. This indicates that the heavy tail characteristics of the lognormal distribution shows the lowest risk, while the gamma distribution lies between the two. This study offers insights for actuaries and risk managers in formulating risk management strategies and assessing capital adequacy for insurance companies. Simulations based on more realistic distributions can help companies design more effective premium-setting and reinsurance policies.

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1. INTRODUCTION

Financial risks arising from certain events are often mitigated by individuals through insurance services. In the broader economy, the insurance industry plays a vital role as an institution that manages and transfers risk from individuals or grups to a company[1]. Insurance companies, in their operations, receive premiums in exchange for providing protection, and in return, are obligated to pay claims when the insured risks occur. However, fluctuations in claim amounts and uncertainty in claim frequency make insurance companies vulnerable to ruin risk [2]. Ruin occurs when the company's reserve funds are no longer sufficient to cover the total incoming claims. This situation can be caused by various factors, such as high aggregate losses during certain periods, premiums that do not reflect the actual level of risk, or an imbalance between premium income and claim obligations [3]. The classical risk process model known as the Cramer-Lundberg model is one of the most widely used approaches in actuarial risk theory to quantitatively understand and measure ruin probability. This model simplifies the dynamics of an insurer's surplus as a stochastic process comprising continuous premium income and randomly occurring claims in both amount and timing [4]. According to this model, the ruin probability can be calculated as a function of parameters such as the claim arrival intensity(λ), the claim amount distribution, premium rate, and initial capital. In practice, fluctuating market conditions and claim patterns that are often far from ideal make analytical approaches of the Cramer-Lundberg model less representative [5]. Therefore, a numerical approach such as Monte Carlo simulation is required, which allows analysis under various scenarios and complex claim distributions, including heavy-tailed distributions commonly found in general insurance [6]. This study focuses solely on the analysis of insurance company ruin probability using the Cramer-Lundberg model enhanced with Monte Carlo simulation. The study is not only theoretical but also provides practical contributions to insurers in performing stress testing of portfolios, estimating minimum capital requirement, and designing pricing and reinsurance strategies that are more adaptive to extreme risks [7].

2. RESEARCH METHODE

This study is a quantitative research employing a computational simulation approach. The main objective is to evaluate the ruin probability of an insurance company using the Cramer-Lundberg model, and to compare the outcomes when claim distributions are modeled using exponential, lognormal, and gamma distributions.

The steps in this research are as follows:

a. Formulation of the Cramer-Lundberg Model

The base model used is:

$$R(t) = u + c(t) - \sum_{i=1}^{N(t)} X_i \, , t \geq 0$$

where,

u = Initial capital

$c(t)$ = Premium income per unit time

$N(t)$ = Number of claims up to time *t*, assumed to follow a poisson process with intensity λ

X_i = Claim size, a positive random variable

b. Claim Distribution Modeling

Simulations are carried out using three types of claim distributions:

1. Exponensial (β) = for light-tailed, small average claims.
2. Lognormal (μ, σ^2) = for large claims with heavy tails
3. Gamma (α, θ) = for flexible distribution shapes

c. Premium Determination

Premiums are determined using the loaded premium principle:

$$c(t) = (1 + \theta)\lambda E(X)$$

where θ is safety loading factor

d. Monte Carlo Simulation

For each parameter combination and claim distribution, simulations are run 10.000 times to estimate the ruin probability:

$$\psi(u) = \frac{\text{Number of simulations with } R(t) < 0}{\text{Total simulations}}$$

e. Ruin Probability Calculation

f. Sensitivity Analysis

Sensitivity analysis is performed on the following variables:

1. Initial capital *u*
2. Claim arrival rate λ
3. Claim distribution parameters
4. Safety loading factor θ

2.1 Classical Risk Theory

Classical risk theory forms the fundamental basis of actuarial science in modelling the surplus of insurance companies. This mode treats the company's cash flow as a stochastic process, under the following assumptions:

a. Premiums are received continuously at a constant rate
b. Claims arrive randomly, following a poisson process
c. Claim sizes follow a specific distribution (e.g., Exponential, Gamma, or Lognormal).

This model was first developed by Filip Lundberg (1903) and later refined by Harald Cramer, and is thus widely knows as the Cramer-Lundberg model. It is used to analyse the ruin probability, which is the likelihood that the insurer's surplus becomes negative within a certain time horizon [5].

2.2 Ruin Probability

Ruin probability is one of the key risk measures in actuarial science. In the Cramer-Lundberg model, it is defined as:

$$\psi(u) = P(U(t) < 0 \text{ for some } t \geq 0 | U(0) = u)$$

This probability depends on various factors including initial capital, premium rate, claim intensity, and the claim size distribution. In some simple cases, such as exponentially distributed claims, analytical solution for $\psi(u)$ exist. However, for more complex distributions, numerical methods or simulations are more commonly used [3].

2.3 Claim Size Distributions

The choice of claims size distributios plays a critical role in risk modelling. Commonly used distributions include:

a. Exponential Distribution
   Frequently used for mathematical convenience. It is memoryless and suitable for modelling small, frequent claims [7].
b. Lognormal Distribution
   More suitable for claim data with high skewness or heavy tails [5].
c. Gamma Distribution
   Offers flexibility for modelling claims with moderate variance [4].

Choosing the right distribution significantly affects the accuracy of ruin probability estimation.

2.4 Monte Carlo Simulation

Monte Carlo simulation is a powerful numerical method used for modelling complex stochastic systems, including those in insurance. In the context of the Cramer-Lundberg risk model, it is used to empirically estimate the ruin probability, particularly when analytical solutions are not feasible due to the nature of the claim distribution [8].

The general steps of the Monte Carlo simulation are as follows:

a. Define the models parameters: initial capital, premium rate, claim intensity, λ, and claim size distribution.
b. Simulate many scenarios of the surplus process $U(t)$ over a specified time horizon.
c. Estimate the ruin probability by calculating the proportion of scenarios that result in negative surplus.

This method accommodates various forms of claim distributions and supports sensitivity analysis on different model parameters [4].

3. RESULT AND ANALYSIS

3.1 Simulation Design

To estimate the ruin probability, a Monte Carlo simulation is conducted with the following parameters:

a. Initial capital $(u) = 100.000$ units
b. Premium income rate $(c) = 1.200$ units per time unit
c. Claim arrival intensity $(\lambda) = 10$ claims per time unit
d. Claim size distribution (X) = Exponential with mean of 10.000 units
e. Simulation time periode $= 50$ time units

f. Number of simulations $= 10.000$ iterations

In each iteration, the surplus process $R(t)$ is simulated from time 0 to 50, and it is observed whether ruin occurs, defined as $R(t) < 0$ for some time t.

3.2 Monte Carlo Simulation Results

Based on 10.000 Monte Carlo simulation iterations, the following result was obtained:

Table 1. Monte Carlo Simulation Results

Number of Iterations	Number of Ruins	Estimated Ruin Probability
10.000	2.134	0.2134

Analysis of Table 1 shows that in 10.000 simulated scenarios, approximately 0.2134 resulted in ruin before time 50. This indicates that even with continuous premium income, randomly arriving claims can still cause a deficit.

3.3 Sensitivity Analysis

To observe the effect of different parameters on ruin probability, variations are made to the initial capital, premium rate, and claim distribution. The following table shows the influence of initial capital,

Table 2. Effect of Initial Capital

Initial Capital (u)	Ruin Probability
50.000	0.4462
100.000	0.2134
150.000	0.0791
200.000	0.0245

From Table 2, it is evident that higher initial capital leads to lower ruin probability. This aligns with the theory that a larger reserve can buffer extreme claim fluctuations.

Based on Table 3, higher premiums increase surpluses more quickly and reduce the likelihood of deficits,

Table 3. Effect of Premium Rate

Premium Rate (c)	Ruin Probability
1.000	0.3147
1.200	0.2134
1.400	0.1258
1.600	0.0632

Furthermore, distributions with heavier tails (such as the lognormal) result in higher ruin probabilities because they allow for rare but high risk large claims.

The simulations results support the theory that ruin probability depends on the balance between initial capital reserves, the rate of premium accumulation, and the intensity and size of claims. The Cramer-Lundberg model performs well as a theoretical representation, but Monte Carlo simulation offers greater flexibility in handling cases with complex claim distributions or realistic scenarios.

The fact that ruin probability can be significantly reduced by adjusting premiums or initial capital highlight the important role of actuaries in designing effective financial strategies for insurance companies.

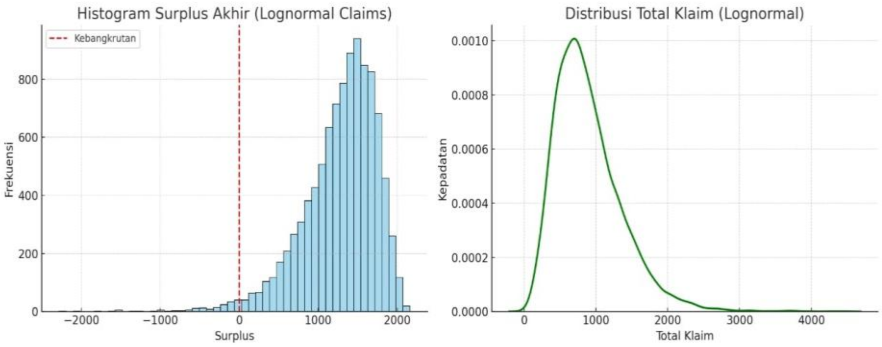

Figure 1. Simulation of the Cramer-Lundberg Model with Lognormal Claim Distributin

In Figure 1, the histogram of the final surplus shows the number of occurrences where the insurance company's surplus is above or below zero. The red line indicates the ruin threshold. Meanwhile, the density plot of total claims illustrates the shape of the total claim distribution within one year, which displays a long right tail (right-skewed) a characteristic of the lognormal distribution.

The comparison of total claim distribution shapes from the three types of distributions can be seen in Figure 2, as follows:

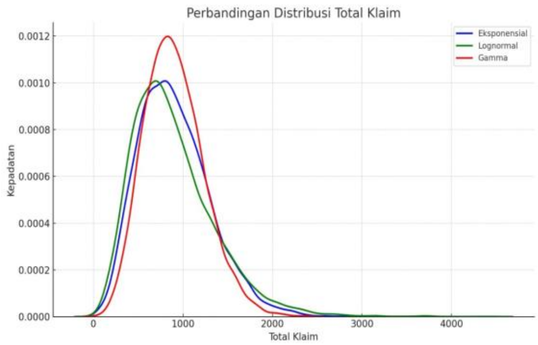

Figure 2. Comparison of Total Claim Distributions

From the graph, it can be seen that the exponential distribution (blue) is narrower, symmetrical, and indicates a small variance. For the lognormal distribution (green), the shape is high right-skewed with a long tail, indicating a higher risk of large claims. In the case of the gamma distribution (red), the distribution lies between the exponential and lognormal, it is slightly skewed but more concentrated than the lognormal. The following are the estimated ruin probabilities based on Monte Carlo simulations for each claim distribution:

Table 4. Estimated Ruin Probabilities Based on Monte Carlo Simulations

Claim Distribution	Ruin Probability
Exponential	0.0056
Lognormal	0.0156
Gamma	0.0008

Overall, Table 4 demonstrates that the choice of claim distribution greatly affects the estimation of ruin probability. Distributions with longer tails (e.g., lognormal) tend to yield higher ruin probabilities, while more concentrated distributions (e.g., gamma) result in more controlled risks.

4. CONCLUSION

Based on the results and simulations conducted, the following conclusions can be drawn:

a. The Cramer-Lundberg model theoretically represents the dynamics of insurance fund surplus by combining continuous premium inflows with randomly occurring claims. It is effective for analysing ruin probability within classical risk theory.

b. Monte Carlo simulations is a highly valuable method for estimating ruin probabilities, especially in situations where analytical solutions are difficult to obtain or when claim distributions do not follow standard forms. In the base case (initial capital $= 100.000$, premium rate $= 1.200$, exponential claims), the estimated ruin probability reaches 21.34% over 50 time units.
c. Sensitivity analysis shows that ruin probability significantly decreases with increased initial capital and premium rates, but increases when using heavy-tailed claim distributions such as lognormal, which have a higher likelihood of extreme claims.

Thus, parameters such as initial capital, premium rate, and claim distribution assumptions are crucial in financial planning and risk management for insurance companies.

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