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Numerical Pricing European Stock Option Based on the Black Scholes Equation Using the Crank-Nicolson Method

ABSTRACT

Accurate option pricing remained a central challenge in quantitative finance, particularly in emerging markets characterized by higher volatility and data uncertainty. This study aimed to estimate the price of European call options based on the Black-Scholes model using the Crank-Nicolson finite difference method. The continuous pricing domain was discretized into uniform spatial and temporal grids, and the numerical results were validated through comparison with the analytical Black-Scholes solution using real stock price data from the Indonesian market. The results showed that the numerical solution converged consistently toward the analytical benchmark as the grid resolution increased, indicating stable and reliable numerical behavior. Overall, the findings confirmed that the Crank-Nicolson method provided an efficient and robust numerical framework for European option pricing, offering a practical balance between computational simplicity and numerical accuracy in moderately volatile financial markets.

Keywords:

European option; Black-Scholes Equation; Crank-Nicolson Method; Finite Difference Method; Numerical Option Pricing.

1. INTRODUCTION

Substantial advances in digital technology have revolutionized modern financial markets through faster information dissemination, increased market efficiency, and the development of increasingly sophisticated financial instruments. These developments have increased trading activity and encouraged innovation in the financial sector, particularly through the use of derivative instruments as an important part of risk management and investment strategies [1][2]. Among these derivative instruments, option contracts play a crucial role due to their sensitivity to changes in the underlying asset price, market volatility, and time to maturity [3][4]. These characteristics make options a widely used instrument by both investors and financial institutions in investment strategies and risk management [5]. Therefore, accurate option pricing is very important in academic research and practical decision-making in finance, especially in emerging financial markets, which are generally characterized by higher volatility and relatively lower market efficiency [6][7].

The theoretical foundation of option pricing was established by Black and Scholes through the formulation of a partial differential equation—commonly referred to as the Black-Scholes equation—that describes the pricing dynamics of European options under idealized market conditions, including constant volatility and frictionless trading [8]. Within this framework, the Black-Scholes model admits a closed-form analytical solution, representing a landmark contribution to the field of financial mathematics. Despite its widespread use as a theoretical benchmark, the practical applicability of the model in real financial markets is subject to several limitations, as its underlying assumptions are frequently violated, particularly in the presence of market incompleteness, time-varying volatility, and the demand for numerically stable and computationally efficient solution methods [9][10].

The limitations inherent in the analytical pricing approach have encouraged to the development of numerical methods for solving the Black-Scholes equation [11]. These numerical techniques become particularly relevant in situations where closed-form analytical solutions are not available or when stable and accurate approximations are required for practical computational applications [12]. From the various numerical schemes, the finite difference method remains one of the most widely used approaches due to its conceptual clarity and ease computational implementation [13]. The Crank-Nicolson method, as a semi-implicit finite difference scheme, has recently attracted considerable attention due to its unconditional stability and second-order accuracy in terms of time and space discretization, which makes it particularly suitable for solving parabolic partial differential equations, such as the Black-Scholes equation [14].

The latest research highlights efforts to improve numerical accuracy in solving the Black-Scholes equation through the development of more advanced computational schemes, including hybrid approaches that combine the Crank-Nicolson method with high-order spatial discretization techniques, such as spline-based formulations [15]. In particular, Roul and Goura (2020) specifically proposed a Crank-Nicolson scheme combined with a sextic spline collocation method and demonstrated higher spatial convergence performance compared to conventional finite difference schemes. However, this increase in numerical accuracy is generally accompanied by increased model complexity and high computational costs, which may limit the practical application of this approach, especially in environments with limited computational resources or in trading and hedging activities that are sensitive to costs [17][18].

It is particularly relevant in the case of developing financial markets such as Indonesia, which are characterized by relatively high stock price volatility and limited availability of consistent historical data. These conditions increase the level of risk associated with the valuation of assets and derivative instruments compared to more mature financial markets [19][20]. The empirical results show that stock market volatility in Indonesia is dynamic and highly influenced by macroeconomic conditions and market structure, thus requiring the application of a stable and reliable modeling approach [21]. However, most of the existing literature focuses on conceptual analysis or market data that is relatively stable, resulting in limited empirical research on the direct application of well-established classical numerical methods in the context of emerging markets [22][23].

Considering the above background, this research aims to apply and validate the Crank–Nicolson finite difference method in solving the classic Black–Scholes equation for determining the price of European-style stock options. We focused on numerical stability, computational efficiency, and ease of implementation, without overcomplicating the model, in line with suggestions in the numerical finance literature. Market data from Indonesia was used to evaluate the practical performance for the proposed method, with the numerical results systematically compared to the Black–Scholes analytical solution as a form of validation, as commonly applied in empirical data-based option pricing research [24]

This research's main contribution was to present a pricing scheme for European options that is clear, numerically stable, easy to implement, and particularly relevant to the context of developing financial markets. With an focus on the use of classic numerical schemes that have been proven effective, this research is expected to serve as a basis for further development towards more complex option pricing models that represent market conditions more realistically.

2. RESEARCH METHOD

2.1 Research Design

In this research, a numerical quantitative approach is applied to estimate the price of European call options based on the classic Black–Scholes model using the Crank–Nicolson finite difference method. The main contribution of this research is computational and numerical, with an emphasis on the direct application of numerical schemes that are strong and have been tested on real stock market data from emerging financial markets.

The Crank–Nicolson method was chosen because it is a semi-implicit finite difference scheme that has unconditional stability and second-order accuracy in time and space discretization, making it well suited for solving parabolic partial differential equations, including the Black–Scholes equation [25]. Select this method to ensure numerical stability while maintaining computational efficiency and ease of implementation.

2.2 Mathematical Model

Mathematical models applied in this research are based on the Black–Scholes partial differential equation that determines the pricing of European call options, which is formulated as follows:

where $V(S,t)$ denotes the option price depends on the underlying asset price S and time t, σ represents the asset volatility, and r denotes the constant risk-free interest rate. Equation (1) is derived under ideal market assumptions and serves as the theoretical foundation for the numerical methods applied in this research [8].

Equation (1) is derived by assumption that the underlying stock price evolves according to a Geometric Brownian Motion (GBM), which is expressed as:

where μ denotes the expected stock price growth rate and W_t denotes a standard Brownian motion [3]. By applying the no-arbitrage principle and transforming the model into the risk-neutral probability measure, the drift parameter μ is replaced by the risk-free interest rate r, resulting in the Black–Scholes formulation [26][27].

For a European call option, the terminal condition at maturity $t = T$ are defined as

where K denotes the strike price and T is the time to maturity of the option. These boundary conditions reflect the theoretical behavior of the European call option value under extreme underlying asset prices [11]. The upper bound S_{max} is chosen large enough to reduce truncation errors at the spatial boundary in the numerical implementation [14].

2.3 Data Source and Parameter Estimation

The empirical data used in this research was obtained from Yahoo Finance, using daily closing stock prices of a large publicly listed company in Indonesia (referred to as PT XYZ). The dataset covers the observation period from January 3, 2025, to December 30, 2025, with a total of 235 daily observations. The model parameters include the most recent stock price (S_0), the strike price (K), the risk-free interest rate (r) obtained from official publications of Bank Indonesia, the annualized volatility (σ) estimated from daily return data, and the time to maturity (T) expressed in years.

The stock price volatility (σ) is computed based on the standard deviation of daily logarithmic returns. The logarithmic return is defined as

where S_t dan S_{t-1} denote the stock closing prices on day-t and the previous day, respectively [28]. The standard deviation of daily returns is computed as:

where n denotes the number of observation days and $\bar{r}$ represents the mean daily return [29]. The annualized volatility is obtained by adjusting the annualized standard deviation of daily returns using the square root of the annual number of trading days in a year ($k = 252$), that is,

This approach a standard procedure in the analysis of financial market volatility based on discrete-time data [28][29].

2.4 Time transformation

To simplify numerical computation, a time-variable transformation is introduced by defining $\tau = T - t$, dan $U(S,\tau) = V(S, T - \tau)$ thereby reformulating the option pricing problem as a forward time-evolution problem. With this transformation, the time derivative can be written as

Substituting equation (8) into equation (1) yields the forward-time formulation of the partial differential equation as

which subsequently became as the basis for the application of the Crank–Nicolson finite difference scheme in this research.

2.5 Spatial Discretization

The spatial domain $[0, S_{max}]$ is discretized into M uniformly spaced grid points with a step size $\Delta S = \frac{S_{max}}{M}$. To approximate the spatial derivatives in equation (9), second-order accurate central finite difference schemes are employed. The first-order spatial derivative at the grid point S_i is approximated as

while the second-order spatial derivative is approximated as:

where U_i^j denotes the numerical approximation of the function $U(S,\tau)$ at the spatial grid point S_i and the j-th time level.

2.6 Crank-Nicolson Time Discretization

Let the time interval $[0, T]$ be divided into N uniform time steps with a step size $\Delta\tau$. Then, apply the Crank–Nicolson scheme to equation (9) and define it as follows:

where $L(\cdot)$ represents the spatial differential operator of the Black–Scholes equation. This discretization process generates a system of linear equations with tridiagonal coefficient matrices, which can then be written in matrix form as follows:

where matrices A and B are tridiagonal matrices that depend on the model parameters and grid sizes, while vector b contains contributions from boundary conditions [13]. System of linear equations generated is effectively solved using Thomas's Algorithm, considering its low computational complexity and good numerical stability for systems with tridiagonal structure [30]. The numerical iteration are performed backward from the maturity time to the initial time. Final value $V(S_0, 0)$ represents the estimated price of the European call option at the present time [13][14].

2.7 Validation and Evaluation

Numerical solutions are validated by comparing them with the Black–Scholes analytical solution for European call options, which is formulated as follows:

with

Where $N(\cdot)$ is the cumulative distribution function of the standard normal distribution. An absolute difference between the numerically calculated option price and the analytical solution is used as an accuracy indicator to assess the convergence rate and accuracy of the applied numerical scheme.

2.8 Computational Implementation

Every numerical calculation in this research was performed using the Python programming language on the Google Colab platform. The computational implementation utilized standard scientific libraries, namely NumPy for numerical operations and Matplotlib for table presentation and graphical visualization. The resulting tridiagonal system of equations was solved using a specialized implementation of the Thomas Algorithm.

3. RESULT AND ANALYSIS

This research aims to evaluate the performance of the Crank–Nicolson finite difference method in pricing European call options within the Black–Scholes framework. The analysis includes empirical data processing, evaluation of numerical convergence behavior, and measurement of accuracy levels through systematic comparison with the Black–Scholes analytical solution.

3.1 Empirical Data and Parameter Estimation

Daily closing stock price data is used as a basis for estimating model parameters. Daily logarithmic returns are calculated based on the ratio of consecutive closing prices presented in Table 1. The analysis results show that stock returns fluctuate around zero without a dominant trend during the observation period, indicating relatively stable short-term price dynamics.

Table 1. Logarithmic Daily Returns of BBCA Stock Prices

t	Date	S_t	S_{t-1}	r_t
0	January 03, 2025	9850	9900	−0.005063
1	January 06, 2025	9675	9850	−0.017926
2	January 07, 2025	9525	9675	−0.015625
⋮	⋮	⋮	⋮	⋮
233	December 29, 2025	8025	8025	0
234	December 29, 2025	8075	8025	0.006211

The Black–Scholes model parameters were obtained through descriptive statistical analysis of logarithmic returns, as presented in Table 2. The resulting annual volatility estimates are within the range commonly found in banking sector stocks in the Indonesian market.

Table 2. Estimated Parameters Used in the Black-Scholes Model

Parameter	Symbol	Value
Last stock price	S_0	8075
Strike price	K	10000
Risk-free rate	r	0.06
Mean daily return	$\bar{r}$	-0.000867
Daily standard deviation	s	0.018502
Annualized volatility	σ	0.293703
Time to maturity	T	0.25

3.2 Numerical results and convergence Analysis

Based on the parameters reported in Table 2, the European call option price is computed numerically using the Crank–Nicolson method and analytically using the Black–Scholes formula. The analytical solution is used as a benchmark to evaluate the accuracy of the numerical method. To measure the degree of agreement between the two approaches, the relative error is defined as

where C_A denotes the option price based on the analytical Black–Scholes solution and C_N denotes the option price computed using the Crank–Nicolson method.

A comparison between the numerical and analytical option prices for various grid sizes in spatial and temporal grid sizes $(M = N)$ is presented in Table 3. The numerical option price s are evaluated at the initial stock price and expressed as $V(S_0, 0)$.

Table 3. Comparison of Numerical and Analytical European Call Option Prices and Relative Errors

$M = N$	Numerical Price	Analytical Pricing	Relative Error (%)
50	62.538028	53.023359	17.944297
100	54.132018	53.023359	2.090887
150	53.599257	53.023359	1.086122
200	53.420202	53.023359	0.748431
250	53.352819	53.023359	0.621349
300	53.127669	53.023359	0.196725
400	53.156822	53.023359	0.251706
500	53.115226	53.023359	0.173257
600	53.061557	53.023359	0.072040

Table 3 shows that increasing grid resolution causes the numerically calculated option prices to approach the analytical Black–Scholes values. When using a coarse grid, the numerical results exhibit relatively large deviations from the analytical values. However, as the grid is gradually refined, the numerical solutions move toward the analytical solutions with a stable and consistent pattern, confirming the good convergence characteristics of the Crank–Nicolson scheme.

3.3 Graphical comparison of Numerical and Analytical Solutions

Figure 1 presents a visualization of the numerical convergence behavior through a comparison between the numerically calculated option prices $V(S_0, 0)$ with the analytical Black–Scholes price C across different spatial grid resolutions.

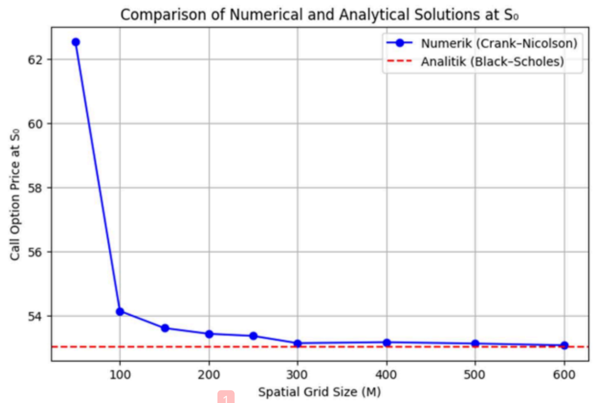

Figure 1. Comparison Between Numerical Crank–Nicolson Solutions and Analytical Black–Scholes Prices for European Call Options

At relatively small values of M, the numerical estimates tends to produce option prices that are higher than those obtained from the analytical solution. As the grid spacing becomes finer, the numerical results gradually approach and eventually nearly coincide with the analytical solution, reflecting the increased accuracy resulting from the more detailed discretization.

3.4 Relative Error Behavior and Spatial Distribution

A comparison of the errors relative to the spatial grid resolution can be observed in Figure 2.

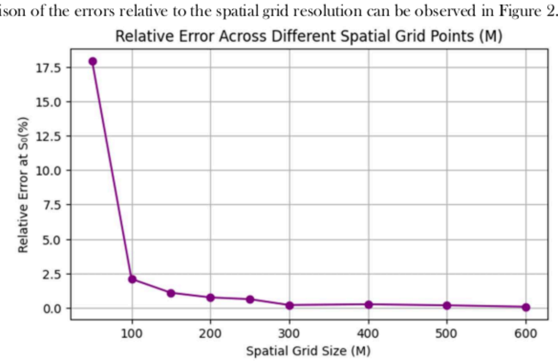

Figure 2. Relative error of the Crank–Nicolson solution with respect to spatial grid size (M)

The results in Figure 2 show that the relative error decreases monotonically with increasing number of spatial grid points, thus confirming the stability and convergence characteristics of the Crank–Nicolson method. At sufficiently fine grid resolution, the relative error is well below 1%, indicating that the method is capable of producing numerical estimates very close to the analytical solution. Furthermore, the distribution of the relative error relative to the underlying stock price is shown in Figure 3.

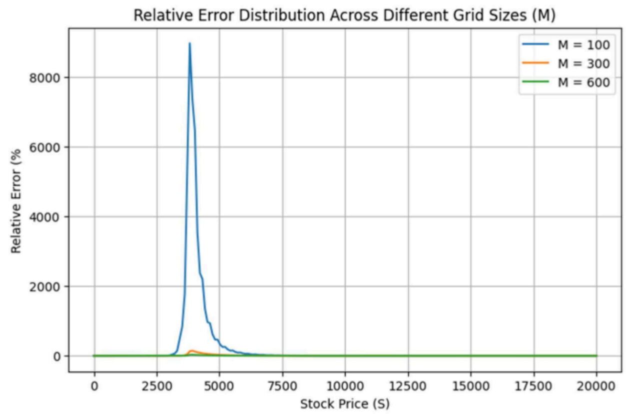

Figure 3. Spatial Distribution of Relative Errors for Different Grid Sizes as a Function of Stock Price

The most significant errors tend to appear around the at-the-money region ($S \approx K$), which is related with local discontinuities in the option payoff function. Outside this range, namely in the in-the-money and out-of-the-money conditions, the relative error tends to be smaller and more stable.

4. CONCLUSION

This research successfully achieved its main objective by applying the Crank–Nicolson numerical method to solve the Black–Scholes equation in calculating the price of a call option (European call option) based on Indonesian stock market data. Through the application of a systematic numerical framework and validation process the analytical solution, the results of this research show that the Crank–Nicolson method is capable of producing stable, convergent, and numerically consistent option price estimates when applied to real market data.

The findings confirm that the Crank–Nicolson method is not only computationally efficient but also practically relevant for emerging financial markets such as Indonesia. Its unconditional stability, ease of implementation, and sufficient accuracy for practical applications make it a reliable alternative to purely analytical approaches as well as to more complex numerical schemes. These results highlight that classical finite difference methods continue to play an important role in empirical quantitative financial modeling.

From a theoretical perspective, this study reinforces the understanding of convergence behavior and numerical stability of semi-implicit schemes in the solution of parabolic partial differential equations arising in derivative pricing. From a practical standpoint, the results may serve as an initial reference for academics and practitioners in developing efficient and easily implementable option valuation systems, particularly in environments with limited computational resources.

Nevertheless, this study is subject to certain limitations, as it is restricted to the classical Black–Scholes model under constant volatility assumptions and does not incorporate dividend payments or more complex option features. Future research is therefore encouraged to extend the modeling framework by considering stochastic volatility, American-style options, or high-frequency data-based approaches to better capture realistic market dynamics. In this regard, the present study is expected to establish a foundation for future advances in numerical method development in derivative pricing for Indonesia and other emerging financial markets.

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